PERGAMON

International Journal of Heat and Mass Transfer 44 (2001) 871–875

International Journal of
HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Law of the wall for two-phase turbulent boundary layers

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Received 3 August 1999; received in revised form 8 March 2000

Abstract

A two-phase logarithmic law of the wall for isothermal bubbly turbulent boundary layer is developed. Total boundary layer liquid turbulent stress is estimated by the sum of bubble induced local stress and shear induced stress. Boussinesq turbulent viscosity approximation is assumed to be valid for both stress components. A proportionality coefficient was introduced to account for inherently non-linear interaction between shear and bubble induced turbulence fields. The two-phase wall law was implemented in CFX4.2 computational fluid dynamics program. A better agreement with experimental data was achieved when the new wall law was employed rather than conventional single-phase law. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Recently, several two-fluid models of isothermal, incompressible, turbulent, bubbly flow were developed and tested [1–3]. Most of the models employed k–ε closure to model Reynolds stress in the liquid phase. All models relied on a single-phase logarithmic law of the wall as a boundary condition. However, single-phase wall law is not valid for turbulent bubbly boundary layer as shown in experiments by Marié et al. [4], Nakoryakov et al. [5,6] and Sato et al. [7]. It was found, that two-phase boundary layer has the same structure as its single-phase counterpart as displayed in Fig. 1. The measured liquid mean velocity profile in the log layer ($y_+ \cong 30$–200) obeyed the logarithmic law:

$$U_+(y_+) = \frac{1}{\kappa'}\ln(y_+) + B', \tag{1}$$

where $U_+ = U_l/U'_w$ is the normalized streamwise liquid velocity, subscript l refers to liquid phase, $y_+ = yU'_w/\nu_l$ is the normalized distance normal to the wall, U'_w is the two-phase frictional velocity, ν_l is the liquid kinematic viscosity. For upward flows, Von Karman (κ') and additive (B') constants in Eq. (1) were found to be functions of mean void fraction, liquid velocity and void fraction shape in the log layer. Measured downward liquid velocity obeyed logarithmic law with single-phase values of B ($B \cong 5.45$) and κ ($\kappa \cong 0.419$) [6].

Preservation of viscous sublayer thickness $y_+^0 \cong 11$ allowed Marié et al. to obtain an expression for the constant B' in terms of κ' and single-phase values of B and κ:

$$B' = y_+^0\left(1 - \frac{\kappa}{\kappa'}\right) + \frac{\kappa}{\kappa'}B. \tag{2}$$

The existence of logarithmic law in bubbly boundary layers allows to assume that (a) similarity hypothesis holds; (b) there exists a local equilibrium between liquid turbulent energy production and dissipation. This fact was exploited by several researchers [7,8] to

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0017-9310/01/$ - see front matter
PII: S0017-9310(00)00128-9

Nomenclature

B	additive constant in the wall law
C_μ	turbulence constant
$C_{\mu b}$	bubble induced turbulence constant
g	gravitational acceleration
F	interfacial force density
J	superficial velocity
k	turbulent kinetic energy
P	mean pressure
$\langle u^2\rangle$, $\langle v^2\rangle$, $\langle uv\rangle$	components of liquid Reynolds stress
U, V	components of mean liquid velocity vector
U_r	local slip velocity
U_w	wall friction velocity
x, y	coordinate components parallel and normal to wall
y_+^0	viscous sublayer thickness

Greek symbols

α	void fraction
β	two-phase correction coefficient
ε	dissipation rate
κ	von Karman constant
κ_1	proportionality coefficient
σ	surface tension
ν	kinematic viscosity
ρ	density
τ	shear stress

Superscript

x	rescaled two-phase wall quantity

Subscript

+	value in wall units
g	gas phase
l	liquid phase
w	wall value

derive two-phase wall law. The total turbulent viscosity was assumed to be the sum of shear and bubble induced components. However, the resulting wall laws were different from logarithmic contrary to experimental findings. The purpose of present research is to derive two-phase logarithmic wall law consistent with experimental observations. In derived logarithmic law, two-phase mixing scales are linear combination of shear and bubble turbulence mixing scales.

2. Derivation of the two-phase wall law

Let us consider an incompressible, isothermal, two-phase, turbulent boundary layer with longitudinal coordinate x and distance from the wall y. The x-component of liquid momentum equation can be cast as [9]:

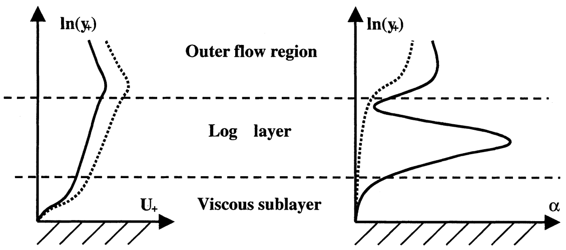

Fig. 1. Schematic of bubbly turbulent boundary layer: solid line represents upward flow; dotted line represents downward flow.

$$\frac{\partial[\rho_l\alpha_l U_l U_l]}{\partial x}+\frac{\partial[\rho_l\alpha_l U_l V_l]}{\partial y}=-\alpha_l\frac{\partial P}{\partial x}+F_x+\rho_l\alpha_l g_x+\frac{\partial}{\partial x}\left(\alpha_l\rho_l\left(2\nu_l\frac{\partial U_l}{\partial x}-\langle u^2\rangle\right)\right)+\frac{\partial}{\partial y}\left\{\alpha_l\rho_l\left[\nu_l\left(\frac{\partial U_l}{\partial y}+\frac{\partial V_l}{\partial x}\right)-\langle uv\rangle\right]\right\}, \tag{3}$$

where U_l and V_l are the longitudinal and lateral components of the mean liquid velocity, $\langle u^2\rangle$ and $\langle uv\rangle$ are the Reynolds stress components, F_x and g_x are the interfacial force density and gravity projections, α_l is the local liquid void fraction and $\langle\cdot\rangle$ denotes phasic ensemble averaging.

Fully developed boundary layer approximation and turbulence predominance is assumed. Interfacial force density is neglected as well. Eq. (3) becomes:

$$\frac{\partial}{\partial y}(-\alpha_l\langle uv\rangle)=0. \tag{4}$$

Introducing turbulent viscosity ν_t and integrating Eq. (4) across the boundary layer yields:

$$\alpha_l\nu_t\frac{\partial U_l}{\partial y}=\frac{\tau_w'}{\rho_l}\equiv U_w'^2, \tag{5}$$

where $\tau_w'=[\alpha_l\rho_l\nu_l(\partial U_l/\partial y)]|_{y=0}$ is the two-phase wall shear stress. Total turbulent viscosity can be written as:

$$\nu_t=\nu_t^{out}+\nu_t^{in}, \tag{6}$$

where ν_t^{out} and ν_t^{in} are the turbulent shear and bubble induced viscosities, respectively. Linear superposition in Eq. (6) is only valid for boundary layer void fraction below 10% [10,11]. To account for non-linearity at higher fractions, a correction will be introduced in the expression for ν_t^{in}. The shear induced viscosity has the form [12]:

$$\nu_t^{out} = \kappa y U'_w. \tag{7}$$

The single-phase value of von Karman constant for length scale in Eq. (7) is consistent with the experimental fact [6], that $\kappa' \cong \kappa$, when $\nu_t^{in} \cong 0$.

A bubble wake induced turbulent viscosity can be estimated as a product of slip velocity U_r and mixing length scale [7]. If bubble size is comparable to boundary layer thickness, its wake mixing length is assumed to be proportional to y. Thus, the following expression for ν_t^{in} is proposed:

$$\nu_t^{in} = \kappa_1 \alpha_{g\,max} U_r y, \tag{8}$$

where $\alpha_{g\,max} = \max(\alpha_g | 30 \leq y^+ \leq 200)$; κ_1 is a non-linearity empirical coefficient. In general, κ_1 depends on the flow character around the dispersed phase. To achieve logarithmic wall law, there must be $\alpha_l \nu_t \propto y$. Since void fraction profile is not known a priori, it is assumed that $\alpha_l \cong 1 - \alpha_{g\,max}$.

Substitution of Eqs. (7) and (8) into Eqs. (5) and (6) yields:

$$\frac{dU_l}{\beta U'_w} = \frac{dy}{\kappa y}. \tag{9}$$

In Eq. (9), scaling coefficient

$$\beta = \left[\left(1 + \frac{\kappa_1 \alpha_{max} U_r}{\kappa U'_w}\right)\left(1 - \alpha_{g\,max}\right)\right]^{-1} \tag{10}$$

is introduced. Solution of Eq. (9) is:

$$U_+^x = \frac{1}{\kappa}\ln\left(y_+^x\right) + B^x, \tag{11}$$

where wall variables y_+^x, U_+^x are calculated using new velocity scale $U_w'^x = \beta U'_w$. Eqs. (1) and (11) are equivalent when $\kappa' = \kappa \beta^{-1}$.

Measured values of β, $\alpha_{g\,max}$ and U'_w in Ref. [4], were used to calculate κ_1 from Eq. (10). Unknown slip velocity for distorted bubbles was evaluated by [13]:

$$U_r = \left[4g\sigma\Delta\rho/\rho_l\right]^{1/4}(1 - \alpha_{max})^{3/4}, \tag{12}$$

where σ is the surface tension and $\Delta\rho$ is the density difference of the phases. Eq. (12) takes into account bubble concentration. Due to the wall void peaking, slip velocity calculated through Eq. (12) is minimal in the boundary in an agreement with [4]. It was found that κ_1 can be approximated by the following formula:

$$\kappa_1 = 4.9453 e^{-40.661 U'_w}, \tag{13}$$

where the frictional velocity is in m/s. Eq. (13) indicates that the relative contribution of bubble wake induced turbulence decreases as shear induced turbulence level increases. Functional dependence $\kappa_1(U'_w)$ also shows that κ_1 depends on other flow parameters, because U'_w, in turn, depends on void fraction, liquid velocity and turbulence level.

Logarithmic law preservation for bubbly flows suggests that dissipation rate of liquid ε_l is equal to turbulence production rate [12]:

$$\varepsilon_l = U_w'^2 \frac{\partial U_l}{\partial y}. \tag{14}$$

Taking into account Eq. (9), Eq. (14) becomes:

$$\varepsilon_l = \frac{U_w'^3 \beta}{\kappa y}. \tag{15}$$

The difference between Eq. (15) and single-phase ex-

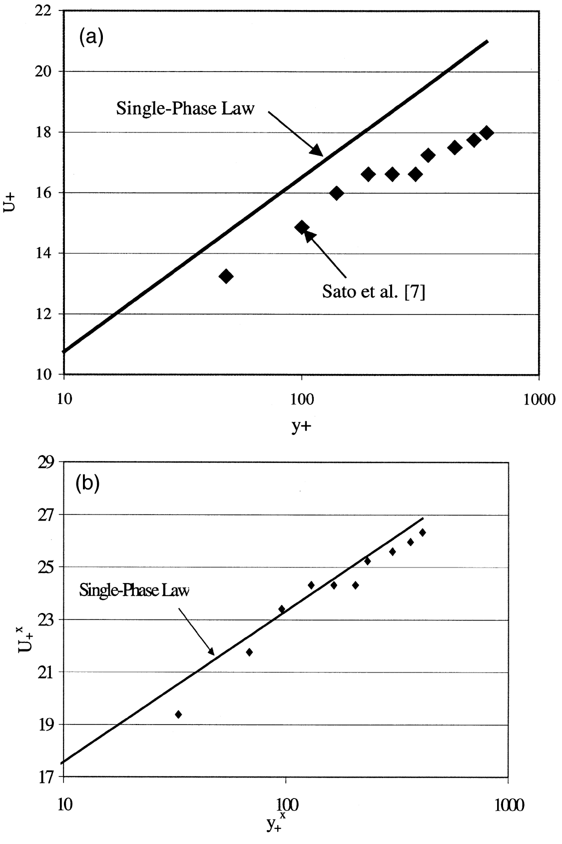

Fig. 2. Mean velocity profile in log layer in: (a) single-phase wall variables; (b) renormalized wall variables.

Table 1
Experimental data used in validation (upward, air–water flow)

Authors	Flow type	d (mm)	J_l (m/s)	$\alpha_{g\,max}$ (%)	U'_w (m/s)
Marie et al. [4]	Bubbly, flat boundary layer	3.5	0.75	2	0.037
	Bubbly, flat boundary layer	3.5	0.75	3.5	0.039
	Bubbly, flat boundary layer	3.5	0.75	6	0.044
	Bubbly, flat boundary layer	3.5	1.0	1.6	0.047
	Bubbly, flat boundary layer	3.5	1.0	3.8	0.049
	Bubbly, flat boundary layer	3.5	1.0	6.8	0.052
Sato et al. [7]	Bubbly, pipe, 26 mm I.D.	4.8	0.58	18.1	0.0463
Nakoryakov et al. [5,6]	Bubbly, pipe, 86.4 mm I.D.	0.8–3	2.05	10	0.0948
	Bubbly, pipe, 86.4 mm I.D.	0.8–3	2.05	9	0.12
	Slug	0.8–3	2.05	2	0.115

pression is that two-phase dissipation time scale is reciprocal to β. Thus, it is determined by not only velocity scale U'_w and length scale y, but also two-phase parameters contained within β.

The standard k–ε model of turbulence models turbulent viscosity as $\nu_t = C_\mu \kappa_l^2 \varepsilon_l^{-1}$, where $C_\mu (= 0.09)$ is empirical constant. Thus, the boundary condition for turbulent energy κ_l is:

$$\kappa_l = \frac{U'^2_w}{\sqrt{C_\mu}} = \text{constant.} \tag{16}$$

Eq. (16) is analogous to its single-phase counterpart.

3. Validation

Eq. (11) was validated against experimental data of [5,7] (see Table 1). An expression for additive constant B^x was derived in [4] on the same grounds as Eq. (2). Fig. 2a and b present logarithmic profiles [7] using conventional and new wall variables. As shown, the rescaled profile is very close to a single-phase law. In the experiment of Ref. [5], calculated β exceeded unity by a value not more than 6%. This is in good agreement with measured value of $\beta \cong 1$.

The new wall law was specifically developed aiming at its eventual implementation in CFD programs. The CFX4.2 [14], a commercially available CFD program with multiphase capabilities, was used to implement the new wall law. The phase coupling was modeled via drag, lift, turbulent dispersion and wall lubrication forces. Turbulence in liquid was described by standard k–ε model with addition of bubble induced turbulent viscosity [7]. Upward bubbly pipe flow experiment of Wang et al. [15] was chosen for validation. Three cases were calculated with liquid superficial velocity $J_l = 0.94$ m/s (Re = 53,500) and air superficial velocities $J_g = 0.1$, 0.27, 0.4 m/s. Two calculations were performed, one with developed two-phase law and the other with conventional single-phase law. Implementation of two-phase wall law resulted in an increased wall friction. Predicted friction velocity was used to non-dimensionalize measured mean liquid velocity and

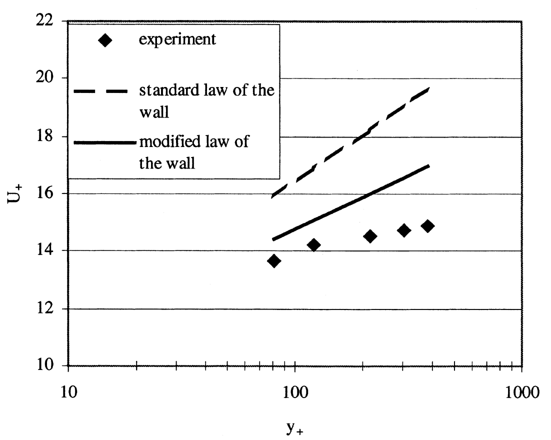

Fig. 3. Mean velocity profile in log layer ($J_g = 0.1$ m/s).

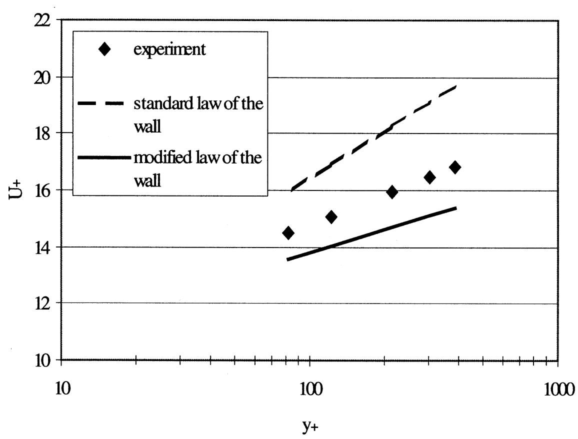

Fig. 4. Mean velocity profile in log layer ($J_g = 0.27$ m/s).

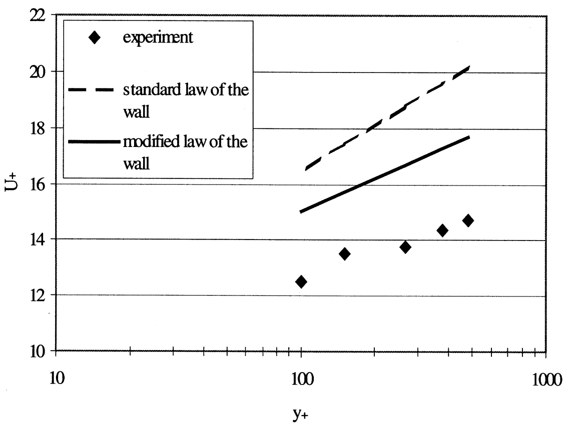

Fig. 5. Mean velocity profile in log layer ($J_g = 0.4$ m/s).

compared with single- and two-phase wall laws as displayed in Figs. 3–5. As shown, experimental profile is closer to the calculations when the developed two-phase wall law was used.

4. Conclusions

Experimental proof of logarithmic law preservation for bubbly boundary layer allows to assume that mixing length approach can be applied to such layer. A modification was introduced to the logarithmic wall law to account for additional bubble induced turbulence in the log layer. The proportionality coefficient accounting for high void non-linearity was introduced and correlated as function of friction velocity. A new wall law was derived where mixing velocity scale is a function of local two-phase parameters. The new law was validated against experimental data of upward bubbly pipe flows and implemented in a CFD code. A better agreement with experimental data was achieved when the two-phase wall law is used over conventional single-phase law.

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